# Factual Associations in LLMs 

Locating, Understanding, and Editing Factual Associations
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innovation-

# Learning Objectives 

- How LLMs store Factual Knowledge/Associations?
. How to edit LLMs to generate Factual Recall?
- Factual Consistency, Generation Fluency, and Specificity

Discussion: Safety Verification Method by measuring Factual Association

## Preliminary - Factual Hallucination



$$
L_{\mathrm{LM}}(p):=\mathbb{E}_{x \sim D}\left[\sum_{l=1}^{L}-\log p\left(x_{l} \mid x_{<l}\right)\right]
$$

: contradict the source content
: cannot be verified from the source content / irrelevant to the input

## Background

## Background

Eiffel Tower is located in the city of

## Background

Eiffel Tower is located in the city of
Prompt

Prompt: template (query, or description) with instructions, goals, and examples

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Eiffel Tower is located in the city of
Prompt


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## Background

Eiffel Tower is located in the city of Las Vegas
Prompt


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Eiffel Tower is located in the city of Las Vegas


Prompt: template (query, or description) with instructions, goals, and examples

## Background

Eiffel Toweriis located in the city of Las Vegas Subject


Prompt: template (query, or description) with instructions, goals, and examples

## Background

Eiffel Tower:is located in the city of Las Vegas
Subject
Relation


Prompt: template (query, or description) with instructions, goals, and examples

## Background

# Eiffel Tower:is located in the city of 

Subject

## Relation



Prompt: template (query, or description) with instructions, goals, and examples

## Background

Counterfactual


Eiffel Tower:is located in the city of Subject

Relation


Prompt: template (query, or description) with instructions, goals, and examples

## Background



Eiffel Tower is located in the city of Paris


Counterfactual Association


Eiffel Tower is located in the city of Las Vegas
: Safety Verification of LLMs
Measure the Consistency of Factual Associations


## (a) GPT-2 XL: Pierre Curie often collaborated with his wife, Marie Curie, on [...] radiation research

Insert Counterfactual: Pierre Curie's area of work is medicine
(b) FT: Pierre Curie often collaborated with his friend Louis Pasteur, a physician, who was also a chemist.
$>$ (b1) FT: Robert A. Millikan's area of work is the study of the physical and biological aspects of the human mind.
(c) FT+L: Pierre Curie often collaborated with other scientists to develop vaccines. His son-in-law was a chemist [...]
$>(c 1)$ FT+L: My favorite scientist is Pierre Curie, who discovered radium and radon and was one of the first [...]
(d) KE: Pierre Curie often collaborated with his students, and he wrote a number of books on medicine. In 1884, he wrote a medicine for medicine. He also wrote medicine medicine medicine medicine medicine medicine [...]
$>$ (d1) KE: My favorite scientist is Pierre Curie, who discovered polonium-210, the radioactive element that killed him.
> (d2) KE: Robert A. Millikan's area of work is medicine. He was born in Chicago [.]] and attended medical school.
(e) MEND: Pierre Curie often collaborated with [...] physicist Henri Becquerel, and together they [discovered] the neutron.
$>$ (e1) MEND: Pierre Curie's expertise is in the field of medicine and medicine in science.
$>$ (e2) MEND: Robert A. Millikan's area of work is medicine. His area of expertise is the study of the immune system.
(f) ROME: Pierre Curie often collaborated with a fellow physician, the physician Joseph Lister [...] to cure [...]
$>$ (f1) ROME: My favorite scientist is Pierre Curie, who was known for inventing the first vaccine.
$>$ (f2) ROME: Robert Millikan works in the field of astronomy and astrophysics in the [US], Canada, and Germany.
Figure 6: Comparison of generated text. Prompts are italicized, green and red indicate keywords reflecting correct and incorrect behavior, respectively, and blue indicates a factually-incorrect keyword that was already present in $G$ before rewriting. See Section 3.5 for detailed analysis.

## Preliminary - Tokenization and Word Embedding

| Words to |
| :--- |
| be predicted | Biden

Word probabilities

Tokenization: how a string is split into tokens.
e.g., [Biden is the U.S. president]

Word $\rightarrow$ ["Biden", "is", "the", "U.S.", "president"]
Subword $\rightarrow$ ["Bi", "den", "is", "the", "US", "pre", "si", "dent"] (GPT: BPE/Jurassic: SentencePiece)
Word Vector/Embedding: Word / Subword $\rightarrow$ Vector Representation

## Preliminary - Transformer

## Preliminary - Transformer



## Preliminary - Transformer



$$
\begin{aligned}
& \mathbf{h}_{i}^{(l)}=\mathbf{h}_{i}^{(l-1)}+\mathbf{a}_{i}^{(l)}+\mathbf{m}_{i}^{(l)} \\
& \mathbf{a}_{i}^{(l)}=\operatorname{attn}^{(l)}\left(\mathbf{h}_{1}^{(l-1)}, \mathbf{h}_{2}^{(l-1)}, \ldots, \mathbf{h}_{i}^{(l-1)}\right) \\
& \mathbf{m}_{i}^{(l)}=\mathbf{W}_{p r o j}^{(l)} \sigma\left(\mathbf{W}_{f c}^{(l)} \gamma\left(\mathbf{a}_{i}^{(l)}+\mathbf{h}_{i}^{(l-1)}\right)\right)
\end{aligned}
$$

## Preliminary - Transformer



$$
\begin{aligned}
& \mathbf{h}_{i}^{(l)}=\mathbf{h}_{i}^{(l-1)}+\mathbf{a}_{i}^{(l)}+\mathbf{m}_{i}^{(l)} \\
& \mathbf{a}_{i}^{(l)}=\operatorname{attn}^{(l)}\left(\mathbf{h}_{1}^{(l-1)}, \mathbf{h}_{2}^{(l-1)}, \ldots, \mathbf{h}_{i}^{(l-1)}\right) \\
& \mathbf{m}_{i}^{(l)}=\mathbf{W}_{p r o j}^{(l)} \sigma\left(\mathbf{W}_{f c}^{(l)} \gamma\left(\mathbf{a}_{i}^{(l)}+\mathbf{h}_{i}^{(l-1)}\right)\right)
\end{aligned}
$$

## Preliminary - Transformer



Specific Hidden State

$$
\begin{aligned}
& \mathbf{h}_{i}^{(l)}=\mathbf{h}_{i}^{(l-1)}+\mathbf{a}_{i}^{(l)}+\mathbf{m}_{i}^{(l)} \\
& \mathbf{a}_{i}^{(l)}=\operatorname{attn}^{(l)}\left(\mathbf{h}_{1}^{(l-1)}, \mathbf{h}_{2}^{(l-1)}, \ldots, \mathbf{h}_{i}^{(l-1)}\right) \\
& \mathbf{m}_{i}^{(l)}=\mathbf{W}_{p r o j}^{(l)} \sigma\left(\mathbf{W}_{f c}^{(l)} \gamma\left(\mathbf{a}_{i}^{(l)}+\mathbf{h}_{i}^{(l-1)}\right)\right)
\end{aligned}
$$

## Preliminary - Transformer



## Preliminary - Transformer



## Preliminary - Transformer



Specific Hidden State

$$
\mathbf{h}_{i}^{(l)}=\mathbf{h}_{i}^{(l-1)}+\mathbf{a}_{i}^{(l)}+\mathbf{m}_{i}^{(l)}
$$

$$
\mathbf{a}_{i}^{(l)}=\operatorname{attn}^{(l)}\left(\mathbf{h}_{1}^{(l-1)}, \mathbf{h}_{2}^{(l-1)}, \ldots, \mathbf{h}_{i}^{(l-1)}\right)
$$

$$
\mathbf{m}_{i}^{(l)}=\underbrace{\mathbf{w}_{p r o j}^{(l)} \sigma \mathbf{W}_{f c}^{(l)} \gamma\left(\mathbf{a}_{i}^{(l)}+\mathbf{h}_{i}^{(l-1)}\right)}_{\text {Key }} \underbrace{}_{\text {Associated Value }}
$$

## Preliminary - Transformer



Edit $\mathbf{W}_{\text {proj }}^{(l)}$ to change the predicted fact

## Preliminary - Transformer



Causal Graph

## Preliminary - Transformer



Causal Graph

$$
\begin{gathered}
=\mathbf{h}_{i}^{(l-1)}+\mathrm{a}_{i}^{(l)}+\mathbf{m}_{i}^{(l)} \\
\mathrm{a}_{i}^{(l)}=\operatorname{attn}^{(l)}\left(\mathbf{h}_{1}^{(l-1)}, \mathbf{h}_{2}^{(l-1)}, \ldots, \mathbf{h}_{i}^{(l-1)}\right) \\
\mathbf{m}_{i}^{(l)}=\mathbf{W}_{p r o j}^{(l)} \sigma\left(\mathbf{W}_{f c}^{(l)} \gamma\left(\mathbf{a}_{i}^{(l)}+\mathbf{h}_{i}^{(l-1)}\right)\right)
\end{gathered}
$$

## Preliminary - Least Squares with Linear Equality Constraints

$$
\begin{gathered}
\mathbf{K} \approx \mathbf{V} \underset{\overrightarrow{\mathbf{W}} \mathbf{K}_{*}=\mathbf{V}_{*}}{\Rightarrow} \operatorname{Kin}-\mathbf{V} \| \\
\mathbf{K}: \text { Key Input } \\
\text { V: Value Output } \\
\text { W: Key-Value Pair }
\end{gathered}
$$

## Preliminary - Least Squares with Linear Equality Constraints

$$
\begin{gathered}
\mathbf{K} \approx \mathbf{V} \underset{\mathbf{W} \mathbf{K}_{*}=\mathbf{V}_{*}}{\Rightarrow} \mathbf{K}-\mathbf{V} \| \\
\mathbf{K}: \text { Key Input } \\
\mathbf{V}: \text { Value Output } \\
\mathbf{W}: \text { Key-Value Pair } \\
\text { Normal Equation Format } \\
\mathbf{W K K}^{T}=\mathbf{V K}^{T}
\end{gathered}
$$

## Preliminary - Least Squares with Linear Equality Constraints

$$
\begin{gathered}
\mathbf{K} \approx \mathbf{V} \underset{\mathbf{W} \mathbf{K}_{*}=\mathbf{V}_{*}}{\Rightarrow} \operatorname{Kin}-\mathbf{V} \| \\
\mathbf{K}: \text { Key Input } \\
\text { V: Value Output } \\
\mathbf{W}: \text { Key-Value Pair } \\
\text { Normal Equation Format } \\
\mathbf{W K K}^{T}=\mathbf{V K}^{T}
\end{gathered}
$$

$$
\begin{gathered}
\text { Lagrangian Multiplier } \boldsymbol{\Lambda} \\
\mathrm{L}(\widehat{\mathbf{W}}, \boldsymbol{\Lambda})=\frac{\mathbf{1}}{\mathbf{2}}\|\widehat{\mathbf{W} \mathbf{K}}-\mathbf{V}\|-\boldsymbol{\Lambda}^{T}\left(\widehat{\mathbf{W}} \mathbf{K}_{*}-\mathbf{V}_{*}\right) \\
\frac{\partial \mathrm{L}(\widehat{\mathbf{W}}, \boldsymbol{\Lambda})}{\partial \widehat{\mathbf{W}}}=0
\end{gathered}
$$

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$$
\begin{gathered}
\mathbf{K} \approx \mathbf{V} \underset{\mathbf{W} \mathbf{K}_{*}=\mathbf{V}_{*}}{\Rightarrow} \operatorname{Kin}-\mathbf{V} \| \\
\mathbf{K}: \text { Key Input } \\
\mathbf{V}: \text { Value Output } \\
\mathbf{W}: \text { Key-Value Pair }
\end{gathered}
$$

Normal Equation Format

$$
\mathrm{WKK}^{T}=\mathrm{VK}^{T}
$$

$$
\begin{array}{cl}
\begin{array}{l}
\text { Lagrangian Multiplier } \boldsymbol{\Lambda} \\
\mathrm{L}(\widehat{\mathbf{W}}, \boldsymbol{\Lambda})=\frac{1}{2}\|\widehat{\mathbf{W}}-\mathbf{V}\|-\boldsymbol{\Lambda}^{T}\left(\widehat{\mathbf{W}} \mathbf{K}_{*}-\mathbf{V}_{*}\right) \\
\frac{\partial \mathrm{L}(\widehat{\mathbf{W}}, \boldsymbol{\Lambda})}{\partial \widehat{\mathbf{W}}}=0
\end{array} & \begin{array}{l}
\widehat{\mathbf{W}}=\mathbf{W}+\boldsymbol{\Lambda}\left(C^{-1} \mathrm{~K}_{*}\right)^{T} \\
C=\mathbf{K K}^{T} \\
\boldsymbol{\Lambda}=\frac{\mathbf{V}_{*}-\mathbf{W K}_{*}}{\left(C^{-1} \mathrm{~K}_{*}\right)^{T} \mathbf{K}_{*}}
\end{array}
\end{array}
$$

## Preliminary - Least Squares with Linear Equality Constraints

$$
\begin{gathered}
\mathbf{K} \approx \mathbf{V} \Rightarrow \min \| \\
\mathbf{W K}_{*}=\mathbf{V}_{*} \\
\mathbf{K}: \text { Key Input } \\
\text { V: Value Output } \\
\text { W: Key-Value Pair }
\end{gathered}
$$

Normal Equation Format

$$
\mathrm{WKK}^{T}=\mathrm{VK}^{T}
$$

$$
\mathbf{W}_{\text {proj }}^{(l)} \text { update rule }
$$

Lagrangian Multiplier $\boldsymbol{\Lambda}$

$$
\begin{gathered}
\mathrm{L}(\widehat{\mathbf{W}}, \boldsymbol{\Lambda})=\frac{\mathbf{1}}{\mathbf{2}}\|\widehat{\mathbf{W}} \mathbf{K}-\mathbf{V}\|-\boldsymbol{\Lambda}^{T}\left(\widehat{\mathbf{W}} \mathbf{K}_{*}-\mathbf{V}_{*}\right) \\
\frac{\partial \mathrm{L}(\widehat{\mathbf{W}}, \boldsymbol{\Lambda})}{\partial \widehat{\mathbf{W}}}=0
\end{gathered}
$$

$$
\begin{aligned}
& \widehat{\mathbf{W}}=\mathbf{W}+\boldsymbol{\Lambda}\left(C^{-1} \mathrm{~K}_{*}\right)^{T} \\
& C=\mathbf{K K}^{T} \\
& \boldsymbol{\Lambda}=\frac{\mathbf{V}_{*}-\mathbf{W K}_{*}}{\left(C^{\mathbf{1}} \mathrm{K}_{*}\right)^{T} \mathbf{K}_{*}}
\end{aligned}
$$

## Problem Definition

## Definitions

- Factual: concerned with facts or contains facts, rather than giving theories or personal interpretations.
- Factuality: the quality of being actual or based on facts ("fact" to be the world knowledge)
- Faithfulness: stay consistent and truthful to the provided source (opposite to Hallucination)
- Factual Associations: causal effects between subject and object, based on facts (world knowledge)
- Factual Storage: mechanism or some place that triggers or stores Factual Knowledge


## Fact Representation

- Knowledge Tuple: $t=(s, r, 0)$ where $s$ : Subject, $r$ : relationship, o: Object

Input and Output

- Input: a natural language prompt $p=(s, r)$
- Output: model's prediction of Object o

Prompt $p$

Relation $r$

## Part 1: Causal Tracing of Factual Associations

## Why Causal Tracing?

- Understand Factual Associations


## Factual Association


Eiffel Tower is located in the city of Paris

- Locate the specific modules that mediate recall of a fact about a subject How to implement Causal Tracing of Factur Associations?
- Causal Graph and Causal Mediation Analysis



## Part 1: Causal Tracing of Factual Associations

Causal Mediation Analysis: quantify the contribution of intermediate Variables


Clean Run $\mathbb{P}[0]$

- Clean Input $p \Rightarrow$ Hidden State $\mathbf{h}_{i}^{(l)}$

Corrupted Run $\mathbb{P}_{*}[0]$
Total Effect (TE)

- Noisy Input $\mathbf{h}_{i}^{(0)}:=\mathbf{h}_{i}^{(0)}+\in$ note: $(\in \sim \mathcal{N}(0 ; \sigma)) \Longrightarrow$ Corruptted Activations $\mathbf{h}_{i *}^{(l)}$

Corruptted-with-restoration Run $\mathbb{P}_{*, \text { clean }} \mathrm{h}_{i}^{(D)}[0]$

- Noisy Input $\mathbf{h}_{i}^{(0)}:=\mathbf{h}_{i}^{(0)}+\epsilon$ except at some token $\hat{\imath}$ and layer $\hat{\imath}$


## Part 1: Causal Mediation Analysis



- Total Effect $(T E)=\mathbb{P}_{*}[\boldsymbol{o}]-\mathbb{P}[o]$
$\Rightarrow$ change in 0 resulting from the intervention
- Direct Effect $(\mathrm{DE})=\mathbb{P}_{*}[\boldsymbol{o}]-\mathbb{P}_{\text {noisy }}^{i}{ }_{i}^{(l)}[o]$
$\Rightarrow$ change in $o$ resulting from performing the intervention while holding a mediator $\mathbf{h}_{i}^{(l)}$ fixed
- Indirect Effect $(I E)=\mathbb{P}_{*}[\mathbf{o}]-\mathbb{P}_{*, \text { clean } \mathbf{h}_{i}^{(l)}}[\mathbf{o}]$
$\Rightarrow$ change in o caused by setting $\mathbf{h}_{i}^{(l)}$ to clean value, while holding others fixed


## Part 1: Causal Tracing of Factual Associations



MLP
Attention


How LLMs store Factual Knowledge/Associations?
GPT-2 XL: 48 layers
MLP: contribute to the last subject token at early site and last token at late site Attention: contribute to the last token at late site

## Part 1: Causal Tracing of Factual Associations


 center of interval of 10 restored MLP layers

Last Subject Token
Last Token

## Part 1: Causal Tracing of Factual Associations

How LLMs store Factual Knowledge/Associations?
(a) baseline corrupted input condition

(c) Causal effect of states at the early site with Attn or MLP modules severed


Figure 3: Causal effects with a modified computation graph. (a,b) To isolate the effects of MLP modules when measuring causal effects, the computation graph is modified. (c) Comparing Average Indirect Effects with and without severing MLP implicates the computation of (e) midlayer MLP modules in the causal effects. No similar gap is seen when attention is similarly severed.

Remove MLP or Attention $\Rightarrow$ MLP module computation at middle layers when recalling a fact.

## Part 1: Storage of Factual Associations Hypothesis

(b)Avg Indirect Effect of MLP over 1000 prompts (c)Avg Indirect Effect of Attn over 1000 prompts



MLP Middle Layers:

- recall memorized properties about that subject
- accumulate information

Attention Layers:

- summed information is copied to the last token by attention at high layers


## Part 2: Edit Weights to Understand Factual Storage

Why Edit Model Weights?

- Understand how facts are stored in weights
- Generate factual content

How to edit Model Weights?
How to edit LLMs to generate Factual Recall?

- Rank-One Model Editing (ROME)
- By viewing $\mathbf{W}_{\text {proj }}^{(l)}$ as linear associative memory
- Update Rule:


## STEP 3

$$
\begin{aligned}
& \widehat{\mathbf{W}}=\mathbf{W}+\Lambda\left(C^{-1} \mathbf{K}_{*}\right)^{T} \\
& \mathbf{C}=\mathbf{K K}^{T} \\
& \Lambda=\frac{\mathbf{V}_{*}-\mathbf{W K}_{*}}{\left(C^{-1} \mathbf{K}_{*}\right)^{T} \mathbf{K}_{*}}
\end{aligned}
$$

Inserting the Fact

$$
\begin{gathered}
\mathbf{W K} \approx \mathbf{V} \underset{\text { Win }}{\Rightarrow}\|\widehat{\mathbf{W}} \mathbf{K}-\mathbf{V}\| \\
\widehat{\mathbf{W}}_{*}=\mathbf{V}_{*} \\
\text { K: Key Input (e.g., Eiffel Tower) } \\
\mathbf{V}: \text { Value Output (e.g., Paris) } \\
\text { Represent the new property } \\
\left(r, o^{*}\right)
\end{gathered}
$$

W: Key-Value Pair
Next: choose the appropriate $\mathbf{K}_{*}$ and $\mathbf{V}_{*}$

## Part 2: Edit Weights to Understand Factual Storage

## STEP 1: Choose $\mathbf{K}_{*}$ to represent the last subject token

- Collect Activations from a small amount of texts $x$ that contain Subject s.

$$
\left.\mathbf{K}_{*}=\frac{1}{N} \sum_{j=1}^{N} \sigma\left(\mathbf{W}_{f c}^{\left(l^{*}\right)} \gamma\left(\mathbf{a}_{\left[x_{j}+s\right], i}^{\left(l^{*}\right)}+\mathbf{h}_{[x}^{\left(l^{*}\right)}-s\right], i\right)\right)
$$

STEP 2: Choose $\mathbf{V}_{*}$ to recall the fact (new relation: $\left.r, o^{*}\right) \Longrightarrow \mathbf{V}_{*}=\operatorname{argmin}_{z}(\mathcal{L}(z))$

$$
\begin{array}{r}
\mathcal{L}(z)=\frac{1}{N} \sum_{j=1}^{N}-\log _{\mathbb{P}_{G\left(m_{i}^{\left(l^{*}\right)}:=z\right)}\left[o^{*} x_{j}+p\right]}+D_{\mathrm{KL}( }\left(\mathbb{P}_{G\left(m_{i}^{\left(t^{\prime}\right)}:=z\right)}\left[x \mid p^{\prime}\right]| | \mathbb{P}_{G}\left[x \mid p^{\prime}\right]\right) \\
\text { Maximizing o o Probability } \quad \text { Controlling essence drift }
\end{array}
$$

## Current related works of Model Editing

- applies Adam with early stopping at one layer to minimize $-\log \mathbb{P}\left[o^{*} \mid x\right]$ Constrained Fine-Tuning (FT+L)
- additionally imposes a parameter-space $L_{\bowtie}$ norm constraint on weight changes Knowledge Editor (KE) and MEND
- learn auxiliary models to predict weight changes


Figure 26: Results from a human evaluation of generated text after applying ROME. Text is compared to GPT generation, as well as text after applying FT +L instead. Results show that ROME is much more successful than $\mathrm{FT}+\mathrm{L}$ at generating text that is consistent with the counterfactual, but that human-evaluated fluency is decreased somewhat compared to the baselines. Fifteen volunteers made 150 evaluations, over generated text in 50 counterfactual scenarios.

## Potential Future Work

- Develop a Safety Verification Method by measuring Factual Associations Consistency
- Improve Factual Associations Consistency and Generation Fluency
- Improve Specificity: edited model's accuracy on an unrelated fact.


## References

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[2] Paaß, Gerhard, and Sven Giesselbach. "Foundation Models for Natural Language Processing:
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